

Extraction of the strong neutron-proton mass difference from the charge symmetry breaking in $pn \rightarrow d\pi^0$

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Abstract

We perform a complete calculation of charge symmetry breaking effects for the reaction $pn \rightarrow d\pi^0$ at leading order in chiral perturbation theory. A new leading-order operator is included. From our analysis we extract δm_N^{str} , the strong contribution to the neutron–proton mass difference. The value obtained, $\delta m_N^{\text{str}} = (1.5 \pm 0.8 \text{ (exp.)} \pm 0.5 \text{ (th.)}) \text{ MeV}$, is consistent with the result based on the Cottingham sum rule. This agreement provides a non–trivial test of our current understanding of the chiral structure of QCD.

1. At the fundamental level of the Standard Model, isospin violation is due to quark mass differences as well as electromagnetic effects [1,2,3]. Amongst the isospin violating effects in hadronic reactions the ones that are charge symmetry breaking (CSB), i.e. that emerge from an interchange of up and down quarks, are of particular interest. Their importance is due to the fact that the neutral–to–charged pion mass difference, which is almost entirely of electromagnetic origin and usually dominates isospin violating hadronic observables, does not contribute here. Therefore, the sensitivity to the quark mass difference $m_d - m_u$ is more pronounced in observables related to CSB.

CSB effects manifest themselves in many different physical phenomena such

as the mass splitting of hadronic isospin multiplets (e.g. $m_n \neq m_p$ [2] and $M_{D^0} \neq M_{D^+}$ [4]), η -decays (for a recent two-loop calculation, see [5] and references therein), the different scattering lengths of nn and pp systems after removing electromagnetic effects in pp scattering (see, e.g. the review article [6]), neutron-proton elastic scattering at intermediate energies [7], hadronic mixing (e.g. $\rho^0 - \omega$ [8] or $\pi^0 - \eta$ [9] mixing) and the binding-energy difference of mirror nuclei known as Nolen-Schiffer anomaly [10]. Recently, experimental evidence for CSB was found in reactions involving the production of neutral pions. At IUCF non-zero values for the $dd \rightarrow \alpha\pi^0$ cross section were established [11]. At TRIUMF a forward-backward asymmetry of the differential cross section for $pn \rightarrow d\pi^0$ was reported which amounts to $A_{fb} = [17.2 \pm 8(\text{stat.}) \pm 5.5(\text{sys.})] \times 10^{-4}$ [12]. In a charge symmetric world the initial pn pair would consist of identical nucleons in a pure isospin one state and thus an interchange of beam and target would have no observable impact so that the cross section should be symmetric. Thus, the apparent forward-backward asymmetry is due to charge symmetry breaking.

A solid theoretical background for investigating CSB effects is provided by chiral perturbation theory (ChPT), the low-energy effective field theory of QCD [13,14,15]. Especially, since electromagnetic and quark mass (strong) effects typically contribute with similar strength, they can only be disentangled within a systematic effective field theory. ChPT has been recently extended to pion production reactions, i.e. to processes with a large initial momentum $p \simeq \sqrt{m_N M_\pi} \simeq 360 \text{ MeV}$, with $M_\pi(m_N)$ the pion (nucleon) mass. The proper way to include this scale in the power counting was presented in Ref. [16] and implemented in Ref. [17], see Ref. [18] for a review article. Within this scheme it turned out to be possible to achieve a quite good theoretical description of s -wave pion production in $pp \rightarrow d\pi^+$ at next-to-leading (NLO) order [19]; p -wave pion production in different channels of $NN \rightarrow NN\pi$ at next-to-next-to-leading (N²LO) order was investigated in Ref. [20]. These developments in our understanding of isospin conserving pion production mechanisms provide a very good starting point for studying isospin violation effects in $pn \rightarrow d\pi^0$ and $dd \rightarrow \alpha\pi^0$. First efforts in this direction were already presented in Refs. [21,22,23] for the $pn \rightarrow d\pi^0$ reaction and in Refs. [24,25,26,27] for $dd \rightarrow \alpha\pi^0$. In this work we improve the theory for the former reaction.

The neutron-proton mass difference is due to strong and electromagnetic interactions [2], i.e. $\delta m_N = m_n - m_p = \delta m_N^{\text{str}} + \delta m_N^{\text{em}}$. As a result of the chiral structure of the QCD Lagrangian, the strength of the rescattering operator in $pn \rightarrow d\pi^0$ depicted in Fig. 1(a) is proportional to a different combination of δm_N^{str} and δm_N^{em} [21,28] (for related work on isospin violation in pion-nucleon scattering see [29]). Thus, the analysis of CSB effects in $pn \rightarrow d\pi^0$ should allow to determine the values of δm_N^{str} and δm_N^{em} individually. This was for the first time stressed and exploited in Ref. [21]. Consistency of these important quantities as determined from $pn \rightarrow d\pi^0$, where they control the strength of

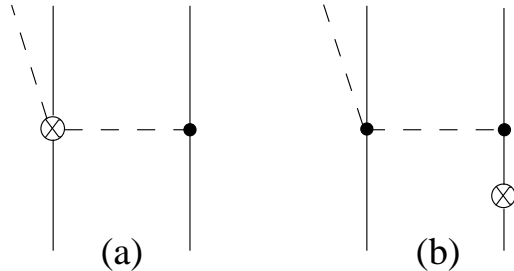


Fig. 1. Leading order diagrams for the isospin violating s -wave amplitudes of $pn \rightarrow d\pi^0$. Solid (dashed) lines denote nucleons (pions). Diagram (a) corresponds to isospin violation in the πN scattering vertex explicitly whereas diagram (b) indicates an isospin-violating contribution due to the neutron–proton mass difference in conjunction with the time-dependent Weinberg-Tomozawa operator (see text for details).

the isospin violating πN scattering amplitude, with results obtained from the neutron–proton mass difference itself [2] employing the Cottingham sum rule [30], would provide a highly non-trivial test of our current understanding of QCD. It was therefore quite disturbing to find that, using the values for δm_N^{str} and δm_N^{em} from Ref. [2], the leading order calculation of the forward-backward asymmetry [21] over-predicted the experimental value by about a factor of 3 — a consistent description would call for an agreement with data within the theoretical uncertainty of 15% for this kind of calculation¹. The evaluation of certain higher order corrections performed in Ref. [21] and in a very recent study [22] did not change the situation noticeably — the significant overestimation of the data persisted.

In this Letter we show that there is one more rescattering operator that contributes at LO. We evaluate this new LO operator and we also recalculate the LO contribution considered in Ref. [21] since the numerical evaluation in that work turned out to be incorrect [32]. The complete LO calculation for $pn \rightarrow d\pi^0$ reveals a very good agreement with the experimental data. Moreover, the resulting contribution is found to be proportional to δm_N^{str} only. Thus, a quantitative understanding of the CSB part of $pn \rightarrow d\pi^0$ promises an alternative method of extraction of this important quantity compared to that used in Ref. [2].

2. The differential cross section of the reaction $pn \rightarrow d\pi^0$ can be expanded into a series of Legendre polynomials $P_i(\cos \theta)$. In the near-threshold region only the first terms are relevant

$$\frac{d\sigma}{d\Omega}(\theta) = A_0 + A_1 P_1(\cos \theta) + \dots, \quad (1)$$

¹ It was shown in Ref. [24] that there is no NLO contribution — thus the theoretical uncertainty of a leading order calculation is expected to be of the order of M_π/m_N .

where θ is the angle between the incident proton and the pion produced and the A_i are functions depending on the different partial wave amplitudes. Due to CSB effects the differential cross section is not symmetric with respect to the replacement $\theta \leftrightarrow \pi - \theta$ and thus A_1 is non-vanishing. The forward-backward asymmetry is defined as

$$A_{fb} = \frac{\int_0^{\pi/2} \left[\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) \right] \sin\theta d\theta}{\int_0^{\pi/2} \left[\frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) \right] \sin\theta d\theta} = \frac{A_1}{2A_0}, \quad (2)$$

where we used Eq. (1) in the last equality. The experiment at TRIUMF was done very close to threshold at $T_{\text{lab}} = 279.5$ MeV, which is equivalent to an excess energy of about 2 MeV or $\eta = 0.17$ — traditionally, the energy for pion production reactions is given in terms of η , the pion momentum in units of the pion mass. At this energy the total cross section $\sigma = 4\pi A_0$ is dominated by the isospin conserving s -wave pion production amplitude. At present, this quantity is known theoretically only up-to-and-including terms at NLO which implies a theoretical uncertainty of the order of 30% for the cross section [19]. Therefore, to minimize the uncertainty of the current study, we use the experimental value for $\sigma(nn \rightarrow d\pi^-) = 252_{-11}^{+5} \cdot \eta$ [μb] extracted with very high accuracy from the lifetime of the pionic deuterium atom², measured at PSI [33]. To convert this number to the reaction of interest here we may use isospin symmetry which gives $\sigma(pn \rightarrow d\pi^0) = \sigma(nn \rightarrow d\pi^-)/2$. Isospin violating effects in this relation are to be expected of natural size and thus will not further be considered. In addition, we include in A_0 also the contribution from the p -wave production. Here we take the results of the N²LO calculation of Ref. [20]. Thus, we get in total $A_0 = 10.0_{-0.4}^{+0.2} \cdot \eta + (47.8 \pm 5.7) \cdot \eta^3$ [μb].

At the energies we consider here, the function A_1 depends on the interference of either an isospin conserving (IC) p -wave and an isospin violating (IV) s -wave amplitude or of an IV p -wave with an IC s -wave. However, only the former piece contributes at leading order. Thus, to the order we are working, one can write

$$A_1 = \frac{1}{128\pi^2} \frac{\eta M_\pi}{p(M_\pi + m_d)^2} \text{Re} \left[\left(M_1^{\text{IC},p} + \frac{2}{3} M_2^{\text{IC},p} \right) M^{\text{IV},s*} \right] \quad (3)$$

where m_d is the deuteron mass and k_π the pion momentum. Here, $M_1^{\text{IC},p}$ and $M_2^{\text{IC},p}$ are the invariant amplitudes corresponding to the isospin conserving p -wave pion production in the $^1S_0 \rightarrow ^3S_1 p$ and $^1D_2 \rightarrow ^3S_1 p$ partial waves and $M^{\text{IV},s}$ is the corresponding amplitude for the isospin violating s -wave production in the $^1P_1 \rightarrow ^3S_1 s$ partial wave. Thus, in the latter amplitude the isovector

² Note that the Coulomb corrections were already removed in the extraction of this quantity from pionic atoms, see, e.g., the review [34].

pion is produced from an isoscalar NN pair ($I_i = 0$). In this work we use the IC p -wave amplitudes of Ref. [20]. As explained in this reference, the contribution $M_1^{\text{IC,p}}$ is quite uncertain and negligibly small. We therefore neglect its contribution in this calculation. The IV s -wave amplitude is discussed in detail below.

3. Our calculations are based on the effective chiral Lagrangian [35,15] which reads

$$\mathcal{L}^{(0)} = N^\dagger \left[\frac{1}{4F_\pi^2} \boldsymbol{\tau} \cdot (\dot{\boldsymbol{\pi}} \times \boldsymbol{\pi}) + \frac{g_A}{2F_\pi} \boldsymbol{\tau} \cdot \vec{\sigma} \cdot \vec{\nabla} \boldsymbol{\pi} \right] N + \dots, \quad (4)$$

for the leading πN interaction terms relevant for our study. The leading isospin-violating terms, generated by the quark-mass difference and hard-photon contributions, are

$$\mathcal{L}_{\text{iv}}^{(0)} = \frac{\delta m_N}{2} N^\dagger \tau_3 N - \frac{\delta m_N^{\text{str}}}{4F_\pi^2} N^\dagger \boldsymbol{\tau} \cdot \boldsymbol{\pi} \pi_3 N - \frac{\delta m_N^{\text{em}}}{4F_\pi^2} N^\dagger (\tau_3 \boldsymbol{\pi}^2 - \boldsymbol{\tau} \cdot \boldsymbol{\pi} \pi_3) N + \dots \quad (5)$$

with $\delta m_N = \delta m_N^{\text{str}} + \delta m_N^{\text{em}}$. The ellipses stand for further terms which are not relevant here. In the equations above F_π denotes the pion decay constant in the chiral limit, g_A is the axial-vector coupling of the nucleon and N ($\boldsymbol{\pi}$) corresponds to the nucleon (pion) field. More precisely, this form of the IV strong and electromagnetic operators is only correct at leading order and neglecting terms with more than two pion fields. The more generic form involves the low-energy constants (LECs) c_5 and f_2 (for precise definitions, see e.g. [28]). Also, beyond LO other strong and electromagnetic LECs will have to be taken into account.

The diagrams that contribute to the amplitude $M^{\text{IV},s}$ at LO are shown in Fig. 1. Diagram (a) corresponds to the rescattering process in which CSB occurs explicitly in the πN scattering vertex due to the last two terms in Eq. (5). In diagram (b) pion rescattering proceeds via the Weinberg-Tomozawa operator (first term in Eq. (4)) which produces an additional isospin violating piece from the mass difference of neutron and proton due to its time dependence as will be discussed later in this section.

In order to understand the interplay of diagram (a) and diagram (b) of Fig. 1 it is sufficient to focus on the πN rescattering vertex on nucleon 1. From the pion production vertex on nucleon 2 we only keep the isospin structure, for the rest is identical for both diagrams. The relevant part of diagram (a) then reads

$$\hat{I}_{(a)} = -i \frac{\delta m_N^{\text{str}}}{4F_\pi^2} \left(\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} + \tau_3^{(1)} \tau_3^{(2)} \right) + i \frac{\delta m_N^{\text{em}}}{4F_\pi^2} \left(\boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} - \tau_3^{(1)} \tau_3^{(2)} \right). \quad (6)$$

We work at leading order in IV. Since we study an IV transition operator, we may therefore treat the external nucleons as identical particles — this is

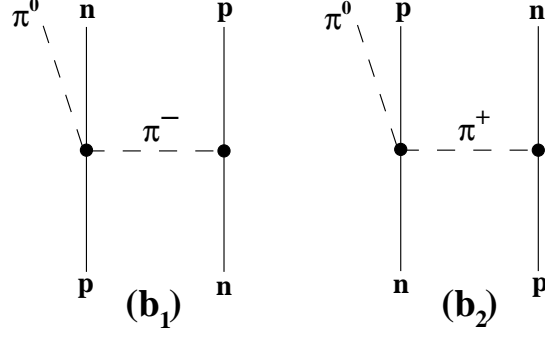


Fig. 2. Leading-order contributions to isospin violation due to the time-dependent Weinberg-Tomozawa operator in the particle basis.

not the case for the diagram (b), where the mass difference of the external particles plays the essential role. The evaluation of the operator Eq. (6) for the isospin violating transition from the isospin zero initial pn state to the isospin zero deuteron state yields

$$\langle I_f = 0 | \hat{I}_{(a)} | I_i = 0 \rangle = \frac{i}{4F_\pi^2} 4 \left(\delta m_N^{\text{str}} - \delta m_N^{\text{em}}/2 \right) . \quad (7)$$

This piece represents the complete rescattering contribution included in Refs. [21,22]. Let us now look more closely at diagram (b) of Fig. 1. The relevant part of the amplitude for this diagram can be most easily calculated in the particle basis as shown in Fig. 2. One gets

$$\langle I_f = 0 | \hat{I}_{(b)} | I_i = 0 \rangle = -\frac{1}{2} (I_{b_1} + I_{b_2}), \quad (8)$$

where I_{b_1} and I_{b_2} are the isospin coefficients corresponding to the diagrams (b₁) and (b₂) of Fig. 2 and the factor $-1/2$ stems from the Clebsch-Gordan coefficients. Note that, since the WT operator involves a time derivative, the corresponding Feynman rule reads

$$V_{WT}^{ab} = \frac{1}{4F_\pi^2} \varepsilon_{abc} \tau_c (q_0 + M_\pi) , \quad (9)$$

with a, b and c Cartesian pion indices and q_μ the four-momentum of the intermediate pion. Due to the explicit appearance of q_0 in V_{WT} , the final expression for diagram (b) of Fig. 1 depends on the neutron-proton mass difference. Indeed, the evaluation of this vertex for the diagrams (b₁) and (b₂) of Fig. 2 yields

$$V_{WT} = \frac{-i}{4F_\pi^2} \begin{cases} \sqrt{2} \left(\frac{3M_\pi}{2} + \delta m_N \right) & \text{for diagram } (b_1), \\ -\sqrt{2} \left(\frac{3M_\pi}{2} - \delta m_N \right) & \text{for diagram } (b_2). \end{cases} \quad (10)$$

Thus, in the isospin violating contribution to Eq. (8) the terms $\propto M_\pi$ cancel while those $\propto \delta m_N$ survive. The non-vanishing isospin matrix element for the diagram (b) of Fig. 1 amounts to

$$\langle I_f = 0 | \hat{I}_{(b)} | I_i = 0 \rangle = \frac{i}{4F_\pi^2} 2\delta m_N. \quad (11)$$

Adding up the contributions of diagrams (a) and (b) we find that the resulting contribution at LO depends on the quark mass contribution to the nucleon mass difference only — the electromagnetic piece vanishes completely:

$$\langle I_f = 0 | \hat{I}_{(a)} + \hat{I}_{(b)} | I_i = 0 \rangle = \frac{i}{4F_\pi^2} 6\delta m_N^{\text{str}}. \quad (12)$$

In comparison with the expression used previously (cf. Eq. (7)) the rescattering operator gets enhanced by about 30%, when standard values $\delta m_N^{\text{str}} = 2$ MeV and $\delta m_N^{\text{em}} = -0.76$ MeV [2] are used.

An alternative method to derive the same result is by using the field-redefined Lagrangian as discussed in Refs [36,37,38] — see also Ref. [39] where unitary transformations are used. In this formulation the pion and nucleon fields are redefined in order to eliminate the first term in the effective Lagrangian in Eq. (5). This allows one to work with nucleons as indistinguishable particles. All terms in the Lagrangian are invariant under this transformation except the ones involving a time derivative such as the Weinberg-Tomozawa operator which generates an additional isospin violating $\pi N \rightarrow \pi N$ vertex $\propto \delta m_N$ that cancels exactly the electromagnetic contribution to this vertex $\propto \delta m_N^{\text{em}}$.

It should be stressed that also in Ref. [21] some effects from the neutron–proton mass difference were included, using the formalism of Ref. [23]. However, these effects appear effectively in the isospin violating πNN vertex and are explicitly in conflict with the chiral structure of QCD. Therefore, they are very different from those discussed above.

For the sake of completeness, we present here the tree-level invariant amplitude $M_{\text{tree}}^{\text{IV},s}$ corresponding to the LO calculation

$$M_{\text{tree}}^{\text{IV},s} = -i \frac{12m_N^2 g_A}{F_\pi^3} \delta m_N^{\text{str}} \int \frac{d\Omega_{p'}}{4\pi} \frac{(\vec{p}' - \vec{p}) \cdot \hat{p}}{(\vec{p}' - \vec{p})^2 + M_\pi^2}, \quad (13)$$

where \vec{p} and \vec{p}' denote initial and final relative momenta of the two nucleons, respectively, and $\hat{p} = \vec{p}/p$. In the calculation we use $F_\pi = 92.4$ MeV and $g_A = 1.32$ (utilizing the Goldberger-Treiman relation). To get the full amplitude $M^{\text{IV},s}$ which enters the observables, $M_{\text{tree}}^{\text{IV},s}$ given above needs to be convoluted with proper NN wave functions in the initial and final states, cf. Appendix A of Ref. [20] for a detailed description. Ideally, one should use wave functions derived in the same framework, namely ChPT. However, up

to now these are only available for energies below the pion production threshold [40]. We therefore adopt the so-called hybrid approach, first introduced by Weinberg [41], i.e. we use transition operators derived within effective field theory and convolute them with realistic NN wave functions [42].

Now we are in the position to discuss the results for the forward-backward asymmetry within the complete LO calculation. Using the values for the parameters specified above and utilizing the NN wave functions from Ref. [42], the result can be presented in the form

$$A_{\text{fb}}^{\text{LO}} = (11.5 \pm 3.5) \times 10^{-4} \frac{\delta m_N^{\text{str}}}{\text{MeV}} . \quad (14)$$

As discussed above, the calculation of the coefficient has a theoretical uncertainty of 15% which is doubled to provide a more conservative estimate. This uncertainty is included in the expression above. We now use the experimental result for A_{fb} [12] to extract δm_N^{str} which yields

$$\delta m_N^{\text{str}} = (1.5 \pm 0.8 \text{ (exp.)} \pm 0.5 \text{ (th.)}) \text{ MeV} , \quad (15)$$

where we added the experimental errors in quadrature. This is the final result of our analysis. At the present stage, the uncertainty in the determination of δm_N^{str} is dominated by the experimental uncertainty for A_{fb} .

In this context let us point out the following: Besides the additional IV contribution discussed in detail above there are other reasons why our result deviates from those of Refs. [21,22] already at leading order. The numerical evaluation of the diagram (a) of Fig. 1 revealed that the value we obtain is significantly smaller than the one found in Ref. [21]. It turned out that the result of that work is too large by a factor of 4 due to an error [32]. The discrepancy of our result to that of Ref. [22] is an accumulation of various effects. First of all in Ref. [22] the isospin conserving s - and p -wave amplitudes are calculated within ChPT up to NLO. Thus, they come with individual uncertainties of 30 % and 15 %, respectively — the uncertainty for the s -wave appears doubled for this amplitude, since it enters squared in A_0 , while the p -wave amplitudes mainly contribute linearly to A_1 — cf. Eqs. (2) and (3). In contrast to this we take the s -wave amplitude directly from data, with a negligible uncertainty and for the p -wave amplitudes the results of Ref. [20], which were calculated to NNLO and are additionally constrained by data. Thus, combining these uncertainties with that for the CSB amplitude in quadrature, a total uncertainty of 50 % arises for the result of Ref. [22]. In addition, the p -wave amplitude with the 1S_0 initial state employed in Ref. [22], which amounts to an enhancement of 50% in the isospin conserving p -wave amplitude in this calculation, is in conflict with the data for $pp \rightarrow d\pi^+$, which calls for a negligible contribution of this partial wave [20]. These effects together — the larger uncertainty of the calculation of Ref. [22] as well as the wrong p -wave amplitude — explain

the discrepancy between our result and that of Ref. [22].

In Ref. [21] also some higher order contributions were calculated, see also [6]. While individually sizeable, the sum of the considered corrections was found to contribute very little to the asymmetry. We re-evaluated these additional pieces and confirmed these findings qualitatively though our results deviate from the ones of Refs. [21,6] quantitatively [43]. In addition, in Ref. [22] some CSB p -wave amplitudes were evaluated. Through an interference with the isospin conserving s -wave they also contribute to the forward-backward asymmetry discussed in this work, however, only at NNLO. It is reassuring that quantitatively these contributions are in line with the power counting estimates given above and thus support our uncertainty estimate.

4. In this work we calculated the CSB forward-backward asymmetry for the reaction $pn \rightarrow d\pi^0$ to leading order in the chiral expansion. We showed that the resulting production operator is driven by that contribution to the neutron-proton mass difference which is coming solely from the quark mass difference, δm_N^{str} . Using the TRIUMF measurement of the forward-backward asymmetry [12] we extracted

$$\delta m_N^{\text{str}} = 1.5 \pm 0.9 \text{ MeV} , \quad (16)$$

where the theoretical and experimental uncertainties are added in quadrature. This number is to be compared with the value for the same quantity extracted from the neutron-proton mass difference — employing the Cottingham sum rule [30] to determine the electromagnetic contribution to the mass difference to $\delta m_N^{\text{em}} = -0.76 \pm 0.3 \text{ MeV}$ [2] —

$$\delta m_N^{\text{str}} = 2.0 \pm 0.3 \text{ MeV} . \quad (17)$$

This value is consistent with a recent determination of the same quantity using lattice QCD [31], $\delta m_N^{\text{str}} = 2.26 \pm 0.57 \pm 0.42 \pm 0.10 \text{ MeV}$ was found, where the uncertainties emerge from statistics, from the input as well as from the chiral extrapolation. We emphasize that the agreement of the various independent extractions provides a highly non-trivial and important test for our understanding of the chiral symmetry and the isospin breaking pattern of QCD, since Eq. (16) is obtained from a reaction where δm_N^{str} is governed by the strength of πN scattering, while Eq. (17) is derived from the neutron-proton mass difference itself. The link between these two apparently very different physical quantities is provided by the symmetry pattern of QCD properly implemented in hadronic matrix elements through chiral perturbation theory.

At present the uncertainty in Eq. (16) is dominated by the experimental error bars — an improvement on this side would be very important. Still, a more refined calculation is also called for since only then one can be confident about the estimated theoretical uncertainty. Work in this direction is in progress.

Acknowledgments

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